(2) Basins are generally shallow relative to their length. Hence, basin oscillations involve standing waves in shallow water. The simplest basin geometry is a narrow rectangular basin with vertical sides and uniform depth. The natural free oscillating period for this simple case, assuming water is inviscid and incompressible, is given by

$$T_n = \frac{2\,\ell_B}{n\,\sqrt{gd}} \qquad \text{Closed basin} \tag{II-7-9}$$

where

T_n = natural free oscillation period

n = number of nodes along the long basin axis (Figure II-7-46)

ℓ_B = basin length along the axis

g = acceleration due to gravity

d = water depth

(3) This equation is often referred to as Merian's formula. The maximum oscillation period T_1 corresponding to the fundamental mode is given by setting $n = 1$ as

$$T_1 = \frac{2\,\ell_B}{\sqrt{gd}} \tag{II-7-10}$$

(4) If the rectangular basin has significant width as well as length (Figure II-7-28), both horizontal dimensions affect the natural period, given by

$$T_{n,m} = \frac{2}{\sqrt{gd}}\left[\left(\frac{n}{\ell_1}\right)^2 + \left(\frac{m}{\ell_2}\right)^2\right]^{\left(-\frac{1}{2}\right)} \qquad \text{Closed basin} \tag{II-7-11}$$

where

$T_{n,m}$ = natural free oscillation period

n,m = number of nodes along the x- and y-axes of basin

ℓ_1, ℓ_2 = basin dimensions along the x- and y-axes

(5) Equation II-7-11 reduces to Equation II-7-9 for the case of a long narrow basin, in which $m = 0$. Further discussion is provided in Raichlen and Lee (1992) and Sorensen (1993). Closed basins of more complex shape require other estimation procedures. Raichlen and Lee (1992) present procedures for a circular basin and approximate solution methods for more arbitrary basin shapes. Defant (1961) outlines a method to determine the possible periods for two-dimensional free oscillations in long narrow lakes of variable width and depth. Locations of nodes and antinodes can also be determined. Usually numerical models are used to properly estimate the response of complex basins.

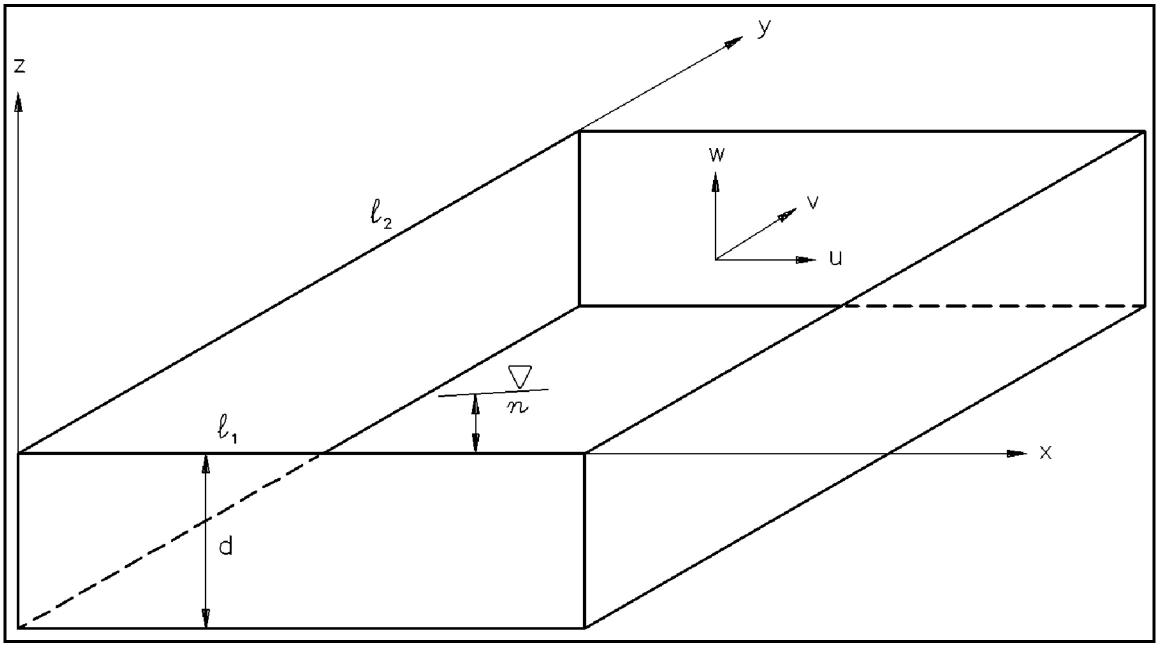


Figure II-7-28. Behavior of an oscillating system with one degree of freedom

(6) Forced oscillations of closed basins can be a concern when the basin is large enough to be affected by moving pressure gradients or strong surface winds, which is generally not true for harbors. Winds can be especially important in large shallow basins, such as Lake Erie. The subject of forced oscillations of closed basins is discussed by Raichlen and Lee (1992).

d. Open basins - general.

(1) Open basins, typically harbors or bays, are most susceptible to oscillations forced across the open boundary. Typical forcing mechanisms include infragravity waves, eddies generated by currents moving past a harbor entrance, and tsunamis (Sorensen 1986). Local seismic activity can also generate oscillations within the basin. Meteorological forces (changes in atmospheric pressure and wind) can initiate oscillations in large bays, but they are not usually a concern over areas the size of a harbor.

(2) The period of a true forced oscillation is the same as the period of the exciting force. However, forced oscillations are usually generated by intermittent external forces, and the period of oscillation is greatly influenced by the basin dimensions and mode of oscillation.

e. Open basins - simple shapes.

(1) In many cases, the geometry of a harbor can be approximated by an idealized, simple shape such as a rectangle or circle. Then the approximate response characteristics can often be determined from guidance based on analytic solutions. Simple harbors are also very helpful for developing an understanding of the general behavior of an open basin.

(2) As with closed basins, the simplest, classical case is a narrow, rectangular basin with uniform depth. The basin has vertical walls on three sides and is fully open at one end. The fundamental mode of resonant oscillation occurs when there is one-quarter of a wave in the basin (Figure II-7-26). The general expression for the free oscillation period in this case is

$$T_n = \frac{4\,\ell_B}{(1+2n)\sqrt{gd}} \qquad \text{Open basin} \tag{II-7-12}$$

(3) The number of nodes in the basin n does not include the node at the entrance. The period for the fundamental mode ($n = 0$) is

$$T_0 = \frac{4\,\ell_B}{\sqrt{gd}} \tag{II-7-13}$$

(4) Maximum horizontal velocities and particle excursions in a standing wave occur at the nodes. Thus there is potential for troublesome harbor conditions in the vicinity of nodes. Some additional useful relationships for standing waves are as follows (Figure II-7-29) (see Sorensen (1986) for details)

$$V_{\max} = \frac{H}{2}\sqrt{\frac{g}{d}} \tag{II-7-14}$$

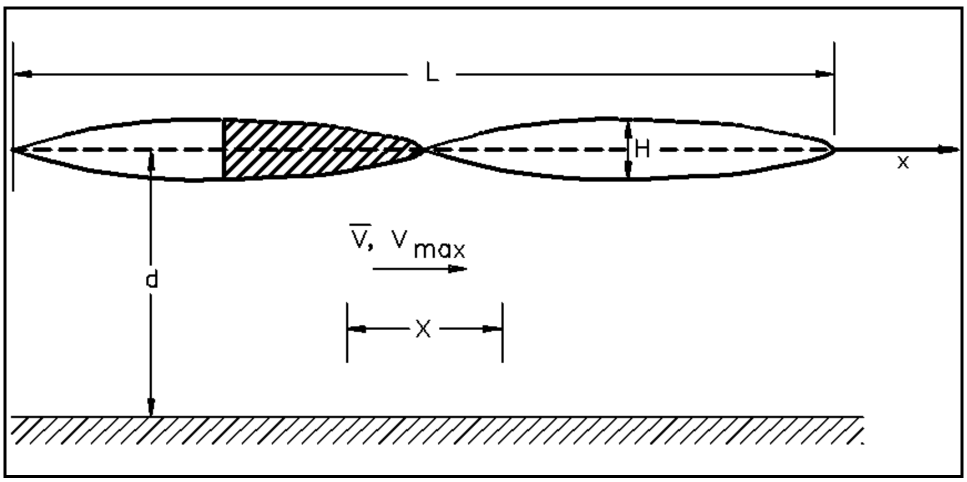


Figure II-7-29. Motions in a standing wave

where

V_{max} = maximum horizontal velocity at a node

H = standing wave height

$$X = \frac{H\,T_n}{2\pi}\sqrt{\frac{g}{d}} \tag{II-7-15}$$

where X = maximum horizontal particle excursion at a node

$$\overline{V} = \frac{H\,L}{\pi\,d\,T_n} \tag{II-7-16}$$

where $\bar{V}$ = average horizontal velocity at a node

(5) For example, when H = 1 ft, T_1 = 200 sec, and d = 30 ft, the maximum horizontal velocity and particle excursion are V_{max} = 0.5 ft/sec and X = 33 ft.

(6) The resonant response of the simple rectangular harbor, presented by Ippen and Goda (1963) in terms of the amplification factor A (assuming no viscous dissipation) and the relative harbor length $k\ell$, where $k = 2\pi/L$, illustrates other important aspects of harbor oscillation. The amplification factor for harbor oscillations is traditionally defined as the ratio of wave height along the back wall of the harbor to standing wave height along a straight coastline (which is twice the incident wave height). The response curve for a long, narrow harbor is given in Figure II-7-30. The left portion of the curve resembles that for the mechanical analogy in Figure II-7-26. Resonant peaks for higher order modes are also shown. The three curves correspond to a fully open harbor and two partially open harbors with different degrees of closure. The $k\ell$ value at resonance decreases as the relative opening width decreases. It is bounded by the value for a closed basin, also shown in the figure. Amplification factors for the inviscid model are upper bounds on those experienced in a real harbor. Because amplification factor decreases at each successive higher order mode, simple analysis methods often focus only on the lowest order modes.

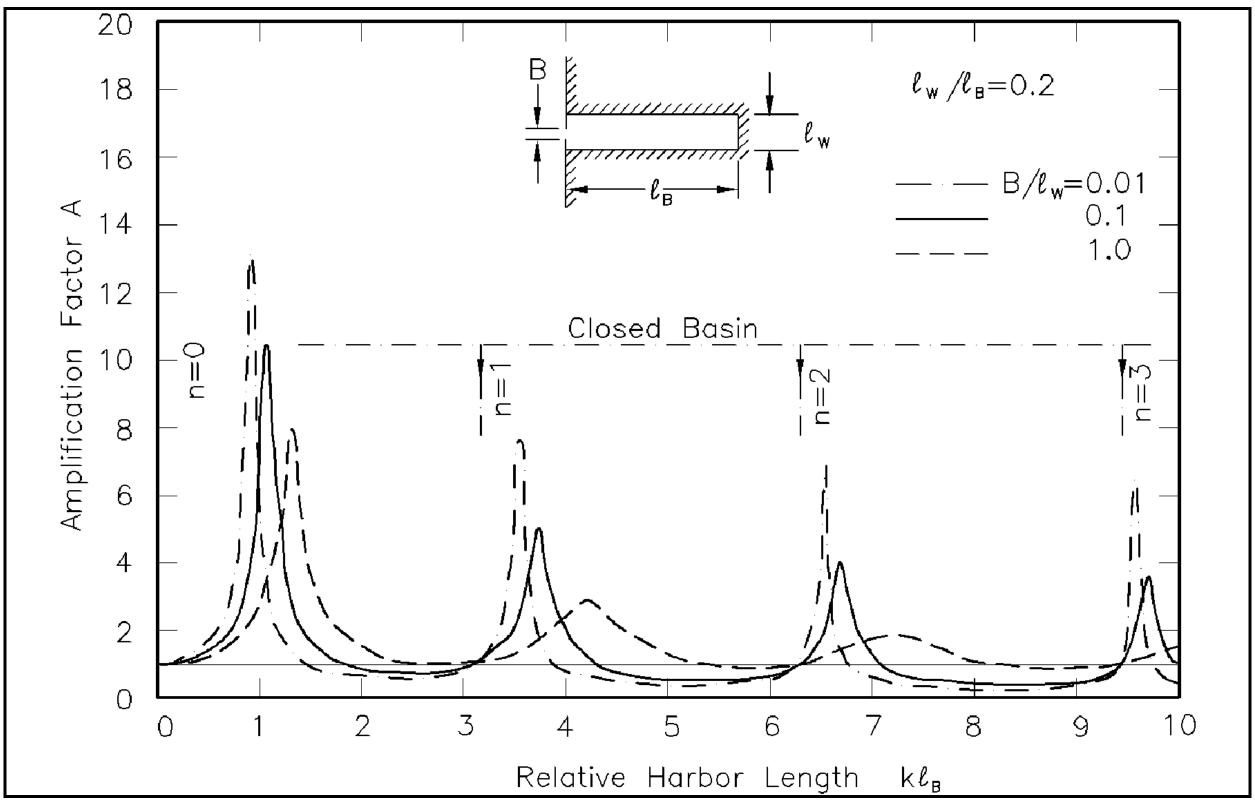


Figure II-7-30. Theoretical response curves of symmetrical, narrow, rectangular harbor (Raichlen (1968); after Ippen and Goda (1963))

(7) Practical guidance for assessing the strength and period of the first two resonant modes in a partially enclosed rectangular harbor with a symmetric entrance is given in Figure II-7-31. A wide range of relative harbor widths (aspect ratios, b/ℓ) and relative entrance widths (d/b) is represented. The $k\ell$ values can be converted to resonant periods by

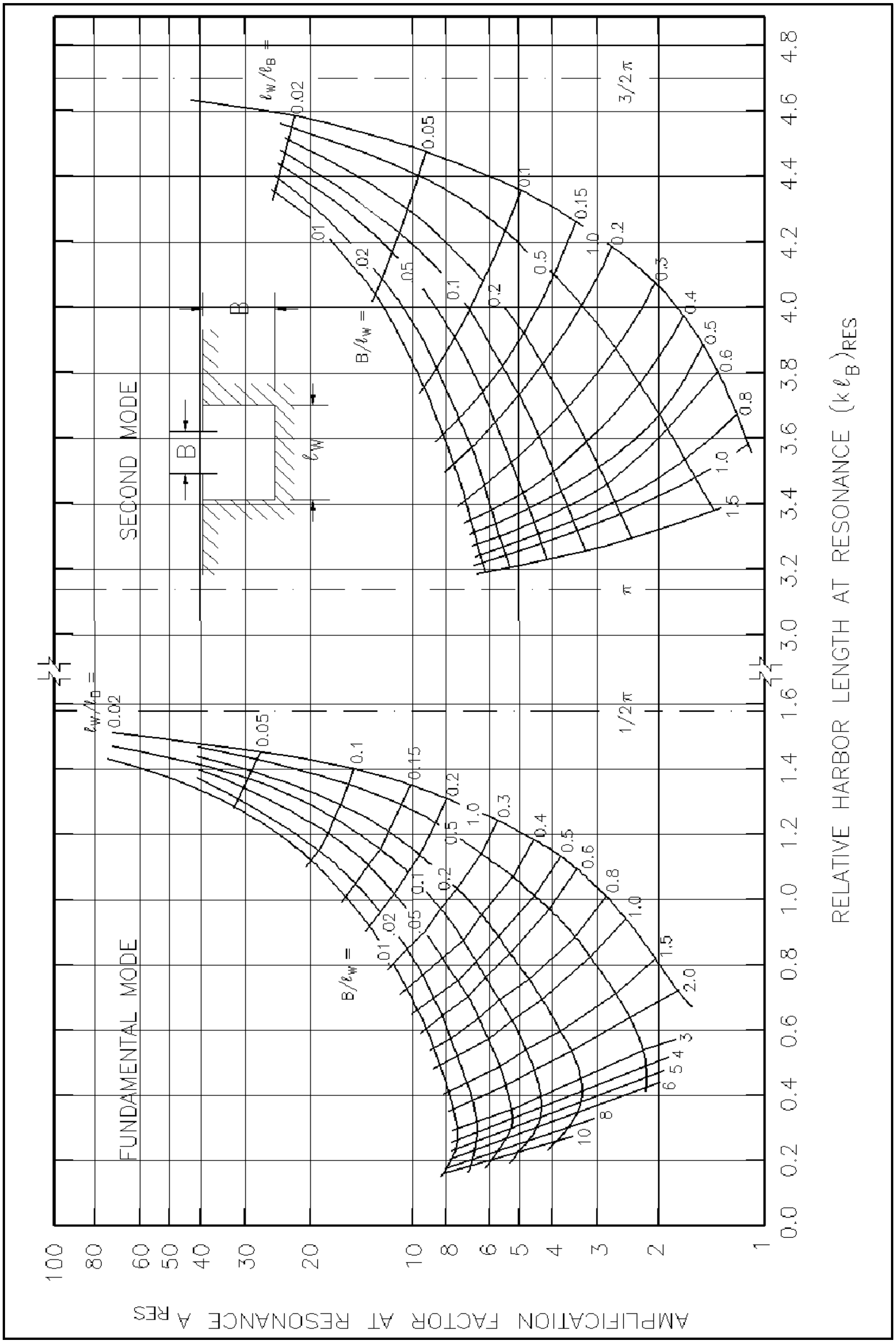


Figure II-7-31. Resonant length and amplification factor of symmetrical rectangular harbor (from Raichlen and Lee (1992); after Ippen and Goda (1963))

$$T = \left(\frac{2\pi\ \ell}{\sqrt{gd}}\right)\ \left(\frac{1}{k\ell}\right) \qquad \text{(II-7-17)}$$

(8) It is interesting to note that the resonant period increases as the aspect ratio increases, even for the fully open harbor. The change is due to the effect of the confluence of the entrance and the open sea. Resonant periods estimated from Figure II-7-30 generally differ from the simple approximation in Equation II-7-12.

(9) The simple guidance has some important limitations relative to real harbors. Amplification factors are upper bounds, since no frictional losses were modelled. Harbors that are not narrow experience transverse oscillation modes that are not represented in the simple guidance. Harbors with asymmetric entrances experience additional transverse modes of oscillation and possible increases in amplification factor (Ippen and Goda 1963). In addition to introducing new resonant frequencies and modified amplification, transverse oscillations change the locations of nodes, as illustrated in Figure II-7-32 based on numerical model calculations.

(10) Harbor oscillation information is available for some other simple harbor shapes. Circular harbors were investigated by Lee (1969) and reviewed by Raichlen and Lee (1992). Zelt (1986) and Zelt and Raichlen (1990) developed a theory to predict the response of arbitrary shaped harbors with interior sloping boundaries, including runup. The dramatic effect of a sloping rather than flat bottom on the behavior of a rectangular harbor is illustrated in Figure II-7-33. The sloping bottom greatly increases A and greatly reduces $k\ell$ values at resonance. For example, $k\ell$ for the second resonant peak drops from 4.2 for the flat bottom case to 2.6 for the sloping bottom case. The first two resonant modes for six symmetric, fully open configurations are given in Figure II-7-34. The A for the first mode is much more affected by the sloping bottom than by the harbor planform. Details are available in Zelt (1986).

f. Open basins - complex shapes.

(1) Real harbors never precisely match the simple shapes; usually they differ significantly. Complex harbors can be analyzed with physical and numerical models. These modeling tools should be applied even for relatively simple harbor shapes when the study has large economic consequences and accurate results are essential. A combination of physical and numerical modeling is usually preferred for investigating the full range of wave conditions in a harbor (Lillycrop et al. 1993).

(2) Physical models generally represent the shorter-period harbor oscillations more accurately. The harbor and immediately surrounding coastal areas are sculpted in cement in a three-dimensional model basin (Figure II-7-35). Breakwaters and other structures that transmit significant amounts of long-wave energy are properly scaled in the model for size and permeability. Currents can be introduced when needed to represent field conditions. Limitations to physical modeling include cost, no direct simulation of frictional dissipation, and the inherent difficulties in working with long waves in an enclosed basin. Special care is required to properly generate long waves. Once generated, they tend to produce undesirable reflections from basin walls.

(3) Numerical models are most useful for very long-period investigations, initial studies, comparative studies of harbor alternatives, and revisiting harbors documented previously with field and/or physical model data. They are also most useful when unusually large areas and/or very long waves are to be studied. For example, numerical models have been used effectively to select locations for field wave gauges (to avoid nodes) and to identify from many alternatives the few most promising harbor modification plans for fine tuning in physical model tests. Lillycrop et al. (1993) suggested that numerical modeling is preferable to

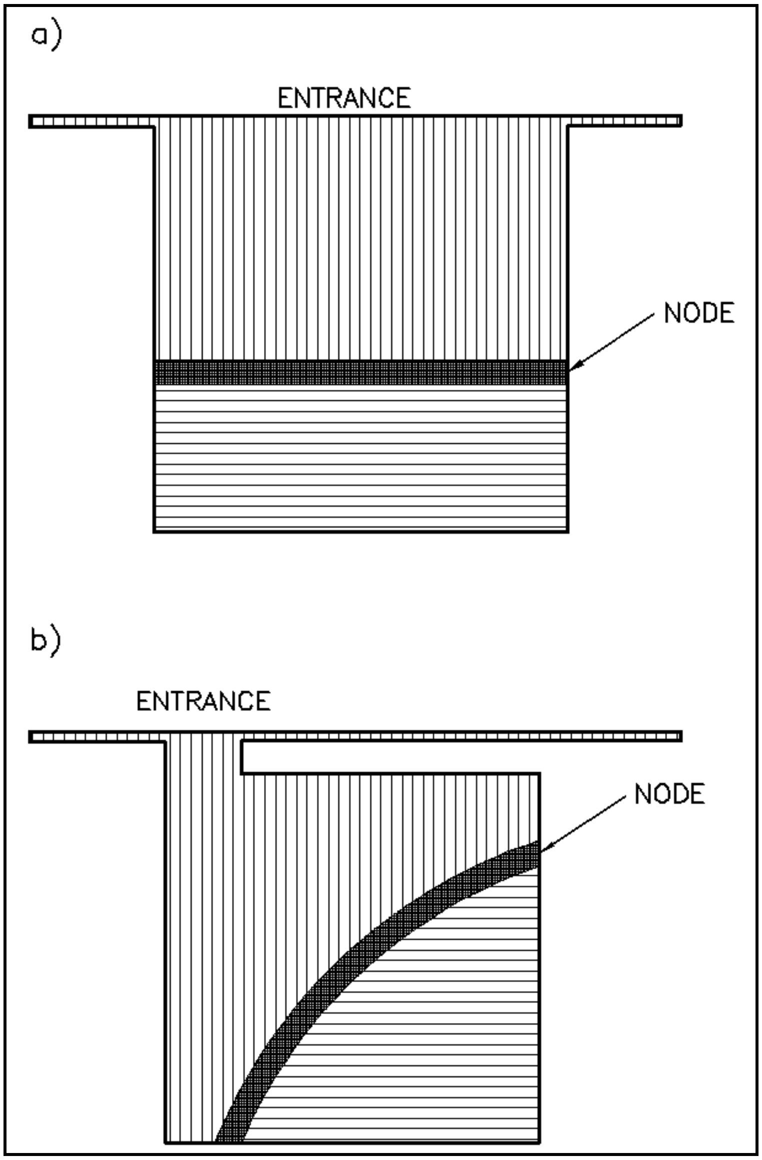


Figure II-7-32. Node locations for a dominant mode of oscillation in a square harbor: a) fully open; b) asymmetric, constricted entrance

physical modeling for periods longer than 400 sec. Both modeling tools can be used effectively for the shorter-period oscillations.

(4) Numerical models can reproduce the geometry and bathymetry of a harbor area reasonably well and estimate harbor response to long waves. Figure II-7-36 is an example numerical model grid. This grid is finer than would normally be required for harbor oscillation studies because it was designed for both wind waves and long waves. Numerical models can be used to generate harbor response curves as in Figure II-7-33 at various points in the harbor. Results for resonance conditions of particular interest can be displayed over the whole harbor to show oscillation patterns. For example, amplification factors and phases calculated with the example grid are presented for five wave periods, corresponding to resonant peaks in the main harbor basin (Figures II-7-37 and II-7-38). Phases are relative to the incident wave. Phase plots are useful because phases in a pure standing wave are constant up to a node and then change 180 deg across the

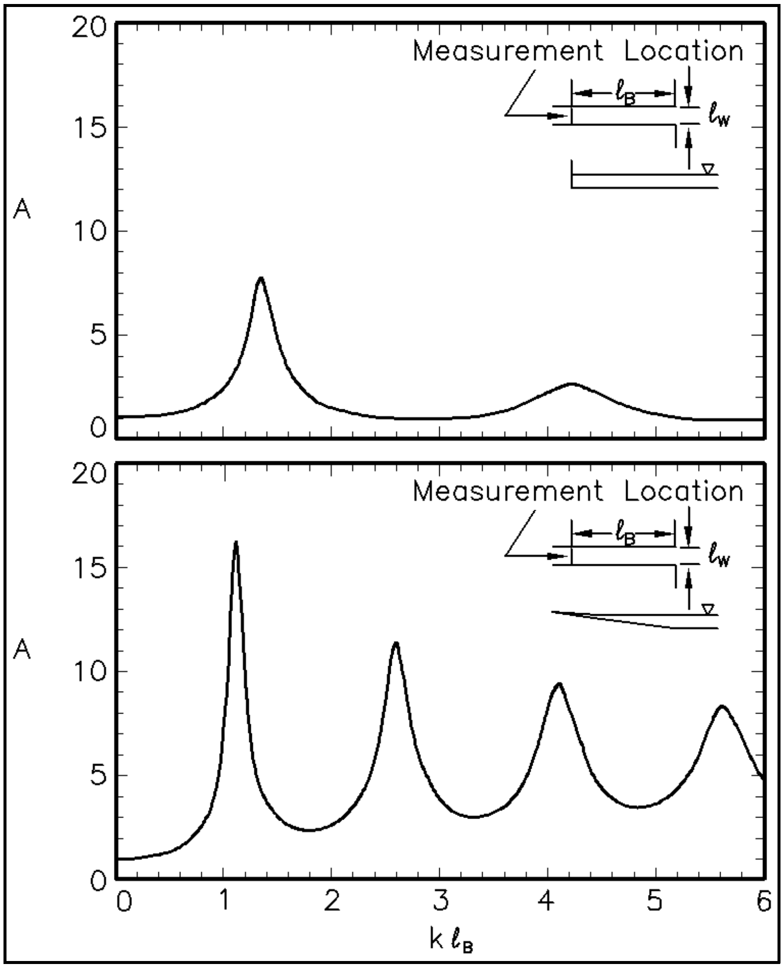


Figure II-7-33. Response curves for rectangular harbor with flat and sloping bottom (Zelt 1986)

node. Thus, phase contour lines cluster together at node locations. Since regions of similar phase, plotted with similar gray shades (or colors on a computer screen), move up and down together, the resonant modes of oscillation can be much more easily visualized. This information about node locations and behavior of resonant modes is quite useful in analyzing harbor oscillations.

(5) Limitations to numerical models relate to approximations in formulating the following items: incident wave conditions, wave evolution during propagation over irregular bathymetry, wave interaction with structures, and harbor boundary features. Numerical model technology is reviewed by Raichlen and Lee (1992) and Abbott and Madsen (1990).

(6) The Corps of Engineers has traditionally used a steady-state, hybrid-element model based on the mild slope equation (Chen 1986; Chen and Houston 1987; Cialone et al. 1991). Variable boundary reflection and bottom friction are included. The seaward boundary is a semicircle outside the harbor entrance. Monochromatic waves are incident along the semicircle boundary. Since the model is linear, results from multiple monochromatic wave runs can be recombined to simulate a spectral response. The model does not account for entrance losses (Thompson, Chen, and Hadley 1993).

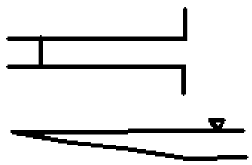
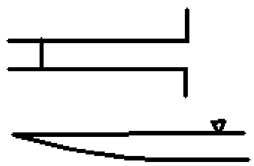
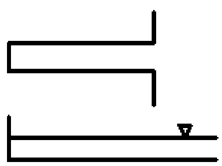
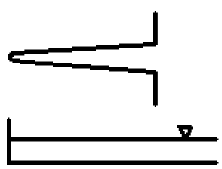
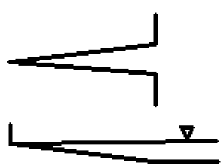
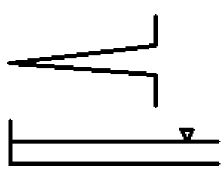
	First Resonant Mode		Second Resonant Mode	
Harbor Geometry	$k\ell_B$	A_{RES}	$k\ell_B$	A_{RES}
	1.089	16.43	2.565	11.45
	1.229	10.96	3.177	7.61
	1.315	7.81	4.182	2.68
	1.696	8.12	4.559	4.32
	1.757	21.85	3.280	32.18
	2.050	8.50	4.926	6.19

Figure II-7-34. Resonant response of idealized harbors with different geometry (Zelt 1986)